Voting with Random Proposers: Two Rounds May Suffice*

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Abstract

This paper introduces the Voting with Random Proposers (VRP) procedure to address the challenges of agenda manipulation in voting. In each round of VRP, a randomly selected proposer suggests an alternative that is voted on against the previous round's winner. In a framework with single-peaked preferences, we show that the VRP procedure guarantees that the Condorcet winner is implemented in a few rounds with truthful voting, and in just two rounds under sufficiently symmetric preference distributions or if status quo positions are not extreme. The results have applications for committee decisions, legislative decision-making, and the organization of citizens' assemblies and decentralized autonomous organizations.

Keywords: strategy-proofness, multi-round voting, majority voting.

JEL Classification: C72, D70, D72.

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1 Introduction

In various collective decision-making scenarios such as committee deliberations, legislative processes, and the governance of proof-of-stake blockchains, members of the corresponding bodies face the challenge of selecting a single alternative from a large set of options. A widely used approach in such contexts is to conduct successive rounds of pairwise majority voting (Rasch, 2000). For instance, legislative bodies often refine bills by sequentially voting on amendments, and negotiating parties in multi-agent resource allocation problems typically converge on a solution through incremental steps. However, with a large number of alternatives, it becomes infeasible to consider all options comprehensively. Moreover, the final outcome is highly sensitive to the subset of alternatives chosen for voting and the sequence in which they are presented. This creates a well-known vulnerability, whereby strategic monopoly agenda setters can manipulate the decision-making process to achieve outcomes aligned with their preferences, as extensively documented in the literature.[1]

To address these challenges, we introduce an iterative majority voting procedure where, in each round, a *randomly selected proposer* from the society suggests an alternative to be voted against the winner of the previous round. We refer to this method as "Voting with Random Proposers" (VRP). The random selection of proposers is designed to decentralize and democratize the agenda-setting process, thus preventing any single agent from dominating the proposal sequence. Our analysis focuses on a simplified framework with a fixed number of voting rounds, one-dimensional alternatives, and a society of agents with symmetric single-peaked preferences with publicly known distribution. Additionally, only the final winning alternative is payoff relevant to the members of the society.

Our main goal is to analyze the strategic incentives created by the VRP procedure for both voters and proposers, while assessing its efficiency in selecting the socially optimal

[1] See the classic contributions by McKelvey (1979), Rubinstein (1979), Bell (1981), Schofield (1983) and the more recent contributions that have extended and applied the potential and limit of agenda power to various settings, Barberà and Gerber (2017), Nakamura (2020) and Ali, Bernheim, Bloedel, and Console Battilana (2023). We refer to Horan (2021) and Rosenthal and Zame (2022) for a recent account of the literature and to Banks (2002) for a review of the earlier literature.

Condorcet winner. We show that the weakly dominant strategy for the proposers in all rounds except the final one is to propose the Condorcet winner, unless their peak lies at the tail of the preference distribution, in which case they propose their own peak. For voters, truthful voting is the weakly dominant strategy, whether they are sophisticated (maximizing continuation utility) or myopic (favoring their current top choice).

As a result, the probability of implementing the socially optimal alternative converges to one after only a few voting rounds. For balanced distributions of the preferences, where the median and mean are sufficiently close, the Condorcet winner is obtained in only two voting rounds, independently of the status quo and the proposers' preferences. Even for arbitrary distributions of preferences in society, the Condorcet winner is selected in two rounds if the status quo is not too extreme. The main intuition is that proposal-makers who are not in the final round face the following trade-off: if they propose a policy at or near their own peak, they risk that the final outcome will shift to the opposite side of the median. This risk can be avoided by proposing the median peak instead.

Our results demonstrate that current iterative voting procedures can be optimized by engaging randomly selected proposal-makers, thereby increasing the likelihood of reaching the Condorcet winner or at least come as close as possible. The VRP procedure has a wide range of potential applications, not only within standard collective decision-making bodies, such as committees and legislative bodies, but also in citizens' assemblies that prepare proposals for city councils or parliaments. In this context, it may be optimal to select two randomly chosen citizen groups to develop proposals, which the assembly would then vote on, advancing the winning alternative to the next decision-making body. Moreover, this procedure could be effectively employed to decide on initiatives in direct democracies and can be easily adapted for decentralized autonomous organizations operating with distributed ledger technologies.

2 Relation to Literature

Since the seminal result of Moulin (1980), it is well known that in the case of single-peaked preferences, the Condorcet winner can be selected by asking agents to announce their peak, adding a number of fixed peaks, and using a suitable generalized median rule. [2] This result has been considerably extended by Border and Jordan (1983), Barberà and Jackson (1994), and Klaus and Protopapas (2020). Unlike their mechanism design approach, our paper focuses on an alternative implementation problem: There is no central authority with commitment, i.e. a mechanism designer. The procedures can only involve proposal-making by agents and voting with equal proposal-making and agenda rights. Such procedures are sometimes called democratic mechanisms (see Gersbach (2009)) and also mirror common practices in parliamentary decision-making in representative democracies and referenda in direct democracies. In particular, we focus on procedures that present agents with two alternatives at a time.

Our paper also relates to the work of Miller (1977), who demonstrates that a Condorcet winner is selected through sophisticated and potentially strategic voting, regardless of the agenda, as long as it includes the Condorcet winner. Similarly, Austen-Smith (1987) shows that strategic voting and sincere voting may be observationally equivalent since agenda-setters will propose options that can defeat the last winning proposal under sincere voting. We extend these insights by showing that the random selection of agenda-setters in an iterative majority voting process creates strong incentives for them to propose the Condorcet winner, even when they are self-interested and strategic. Additionally, we demonstrate that this procedure is strategy-proof.

Legislative bargaining models commonly feature random proposer selection and iterative majority voting on proposals, a framework originating from the seminal work of Baron and

[2] Moulin (1980) has shown that any strategy-proof social choice function on the domain of single-peaked preferences is a generalized median-rule that selects the median of the voters' ballots and $n-1$ fixed ballots, where n is the number of voters. However, not all generalized median rules select the Condorcet winner.

Ferejohn (1989). While much of the subsequent literature has focused on bargaining with an exogenous status quo[3], another line of research, notably Anesi and Seidmann (2014), explores dynamic bargaining models with an endogenous status quo, where the status quo is determined by the outcome of the bargaining in the previous round.[4] Our model features several important deviations from these frameworks, including a finite number of voting rounds, a one-dimensional policy space[5], and a single payoff-relevant outcome. Within this context, our model can be classified as a spatial legislative bargaining model featuring an endogenous status quo and a finite horizon. We obtain moderation due to the threat of future polarization: If I know that my ideological opponent will make a proposal tomorrow, I have an incentive to constrain her through my proposal today. In the presence of a decisive median voter, the way to impose such a constraint is to propose a policy that appeals to the median.

Our paper also relates to the literature on strategic polarization. Kalai and Kalai (2001) demonstrate that strategic behavior within aggregation games tends to foster polarization. In comparison, our work illustrates that well-crafted institutional mechanisms, such as the Voting with Random Proposers procedure, can promote moderation and facilitate convergence toward compromise, more exactly the Condorcet winner. The intuition behind this contrast is the following. Polarization arises when each player directly influences the aggregate outcome. VRP, however, counters this by removing direct aggregation and instead using randomized sequential proposals with majority voting. In another work, Eraslan, Evdokimov, and Zápal (2020) show that equilibrium outcomes evolve gradually through repeated majority voting, but the process may take many rounds. In comparison, our investigations demonstrate that randomizing proposers aligns incentives so that the Condorcet winner emerges almost immediately, often within just two rounds. Thus, while the former relies on

[3]See Eraslan and Evdokimov (2019) for a review.

[4]This literature has been recently surveyed in Eraslan, Evdokimov, and Zápal (2022).

[5]In this respect, our paper relates to the literature on spatial bargaining, including Baron (1991), Banks and Duggan (2000), Kalandrakis (2016), and Zapal (2016).

gradual stabilization through voting dynamics, the latter achieves rapid convergence through institutional design of the proposal stage.

Our research on voting with random proposers over a large set of alternatives also relates to the literature on algorithmically determined and iterative voting procedures with a random component in various policy spaces, as developed by Airiau and Endriss (2009), Goel and Lee (2012) and Garg, Kamble, Goel, Marn, and Munagala (2019), among others. We add to this literature by studying a simple procedure to implement the Condorcet winner.

Finally, when designing iterative voting procedures with endogenous proposal-making, it is crucial to identify alternatives in each round that are closer to the Condorcet winner.[6] In this sense, our work contributes to the literature by Callander (2011), Riboni and Ruge-Murcia (2010), and Barberà and Gerber (2022), which examines how to identify desirable policies to put to a vote. We demonstrate that the sequential random selection of proposers plays a pivotal role in guiding proposers toward making socially superior proposals.

3 The Model

Consider a society that consists of a continuum of agents with mass one, each of them privately informed about their type $\theta \in [0,1]$. Throughout this paper, we make the assumption that the agents' types are distributed according to a cumulative distribution function F allowing for a density f and satisfying either:

$$\int_0^{\theta_\mu} \big(1 - F(\theta_\mu + t) - F(\theta_\mu - t)\big)\, dt \;\geqslant\; 0, \quad \text{if } \theta_\mu \leqslant 0.5,$$

or

$$\int_0^{1-\theta_\mu} \big(1 - F(\theta_\mu + t) - F(\theta_\mu - t)\big)\, dt \;\leqslant 0, \quad \text{if } \theta_\mu > 0.5,$$

[6] We use reaching the Condorcet winner as the objective of designing voting procedures.

where θ_μ denotes the median of the distribution. Note that this condition also implies, on the one hand, $\mathbb{E}(\theta) \geqslant \theta_\mu$ if $\theta_\mu \leqslant \frac{1}{2}$ and, on the other hand, $\mathbb{E}(\theta) \leqslant \theta_\mu$ if $\theta_\mu > \frac{1}{2}$.[7]

Some prominent distributions which satisfy this condition are the uniform $U([0,1])$, all Beta distributions $\mathrm{Beta}(\alpha, \beta)$, the truncated Normal and the truncated Logistic distribution (where truncated means that we draw (and possibly independently redraw) from the more general distribution but only keep values falling in $[0,1]$).

A more classical assumption which implies this somewhat technical condition is to ask for the agents' types to follow an absolutely continuous distribution $\theta \sim F$ with nowhere-zero density function $f > 0$ and median θ_μ such that $1 - F(\theta_\mu - x) - F(\theta_\mu + x)$ is of the same sign for all $x \in [0,1]$.

The distribution of the types is public knowledge. The utility of an agent of type θ from an alternative $x \in [0,1]$ is given by $u_\theta(x) = -(x-\theta)^2$, therefore it has a unique maximum at θ and is symmetric around θ.

We define a function $c : [0,1] \to [0,1]$, $x \mapsto \min\{\max\{2\theta_\mu - x, 0\}, 1\}$. Thus, the function gives for each x the alternative with the following property: the farthest positioned alternative in $[0,1]$ that is just as much or more preferred than x by the society. Note in particular that whenever $x \in (\max\{0, 2\theta_\mu - 1\}, \min\{2\theta_\mu, 1\})$, we have $F\left(\frac{x+c(x)}{2}\right) = F(\theta_\mu) = \frac{1}{2}$.

Furthermore, we define a lower and upper threshold for the types as follows

$$\underline{\theta} = \max\{0, \{\theta < 2\theta_\mu - 1 : \int_\theta^1 u_\theta(v) f(v) dv = u_\theta(\theta_\mu)\}\},$$
$$\bar{\theta} = \min\{1, \{\theta > 2\theta_\mu : \int_0^\theta u_\theta(v) f(v) dv = u_\theta(\theta_\mu)\}\}.$$

Intuitively, the lower threshold type $\underline{\theta}$ has the same expected utility from final winners larger than the own peak as the utility from the median peak. Similarly, the upper threshold type $\bar{\theta}$ has the same expected utility from final winners smaller than the own peak as the utility from the median peak. In right-skewed distributions, $\bar{\theta} = 1$, and in left-skewed distributions,

[7]This can be seen by writing out the term $\mathbb{E}(\theta) - \theta_\mu$ as an integral.

$\underline{\theta} = 0$ by definition. Observe that for distributions of the types for which the expected utility of the two most extreme types over all possible final winners in $[0, 1]$ is smaller than the utility from the median peak, the upper and lower threshold types are 1 and 0, respectively.

Voting with Random Proposers

The society uses a multi-round voting procedure called Voting with Random Proposers to implement an alternative from the interval $[0, 1]$ in a publicly known fixed number of voting rounds T. Suppose that initially, there is a status quo alternative $q^1 \in [0, 1]$. In round $t = 1$, an agenda-setter of type θ_{s^1} is randomly drawn from the distribution F and makes a proposal $p^1 \in [0, 1]$. Then, a majority-vote between p^1 and q^1 takes place, where ties are resolved in favor of the proposal. All voters in the society take part in each round of voting. The winner of the first voting round w^1 becomes the status quo in the next round, i.e. $w^1 = q^2$. In round $t = 2$, a new agenda-setter is randomly drawn and the process is repeated. The procedure ends at round T and the winner of the last round w^T is implemented. Agenda-setters and voters are assumed to be strategic, i.e. they would choose a proposal or vote in favour of alternatives in order to maximize the proximity of the final winner to their type.

Equilibrium concept

To analyze the dynamic voting game, we use the equilibrium concept of Subgame Perfect Nash Equilibrium (SPNE), with the refinement of stage-undomination (Baron & Kalai, 1993), i.e. given future play prescribed by the strategy profile, no player ever plays a weakly dominated action in any voting subgame. Stage-undomination also allows to pin down the voting behavior of individuals who are not critical, which is not constrained by subgame perfection. Under stage-undomination, individuals always vote for the alternative that maximizes their continuation utility, as if they were critical. For tractability, we use a continuum of voters and apply stage-undomination as if we had a finite number of agents. Henceforth, we call an SPNE satisfying the refinement of stage-undominated strategies simply an equilibrium.

4 Optimal Proposal-Making and Truthful Voting

In this section, we study the optimal proposal of the proposers, depending on their type, the distribution of preferences, and the status quo, as well as the outcome of the VRP procedure. As we show in the theorem below, the equilibrium strategy for all proposers in rounds $t \in \{1, 2, \ldots, T-1\}$ is identical, and they either propose their own type or the Condorcet winner. In the last round of voting, the agenda-setter proposes either their own type or the farthest positioned alternative that wins in a pairwise vote against the status quo.

Theorem 1. *The optimal proposal under Voting with Random Proposers is given as follows: in rounds $t \in \{1, 2, \ldots, T-1\}$*

$$p^t = \begin{cases} \theta_{s^t} & \text{if } \max\{q^t, \theta_{s^t}\} < \underline{\theta} \text{ or } \min\{q^t, \theta_{s^t}\} > \bar{\theta}, \\ \theta_\mu & \text{else.} \end{cases} \quad \text{and } p^T = \begin{cases} \min\{\theta_{s^T}, c(q^T)\} & \text{if } q^T \leqslant \theta_\mu, \\ \max\{\theta_{s^T}, c(q^T)\} & \text{if } q^T > \theta_\mu. \end{cases}$$

Truthful voting is a weakly dominant strategy. The probability of implementing the Condorcet winner for $T \in \{2, 3, \ldots\}$ is given by

$$Prob(w^T = \theta_\mu) = \begin{cases} 1 - F(\underline{\theta})^{T-1} & \text{if } \theta_\mu \geqslant 0.5, \\ 1 - (1 - F(\bar{\theta}))^{T-1} & \text{if } \theta_\mu < 0.5. \end{cases}$$

Proof. See Appendix A. □

Proposers face a trade-off between proposing their own type to maximize their utility and proposing the Condorcet winner to insure themselves against unfavorable final winners. The closer a proposer's type is to the Condorcet winner, the stronger the incentive to propose it. The share of such voters increases with the symmetry of the distributions of the types. However, in sufficiently skewed distributions, the Condorcet winner is positioned closer to one extreme, leading proposers with preference peaks at the tail of the distribution to favor their own type. This is the case, because the potential benefit from having all subsequent proposers positioned at the tail outweighs the insurance gained by proposing the Condorcet winner, which is nearly as disliked as the extreme opposite alternatives. The optimal strategy of the proposers is illustrated in Figure 1, depending on the distribution of the types.

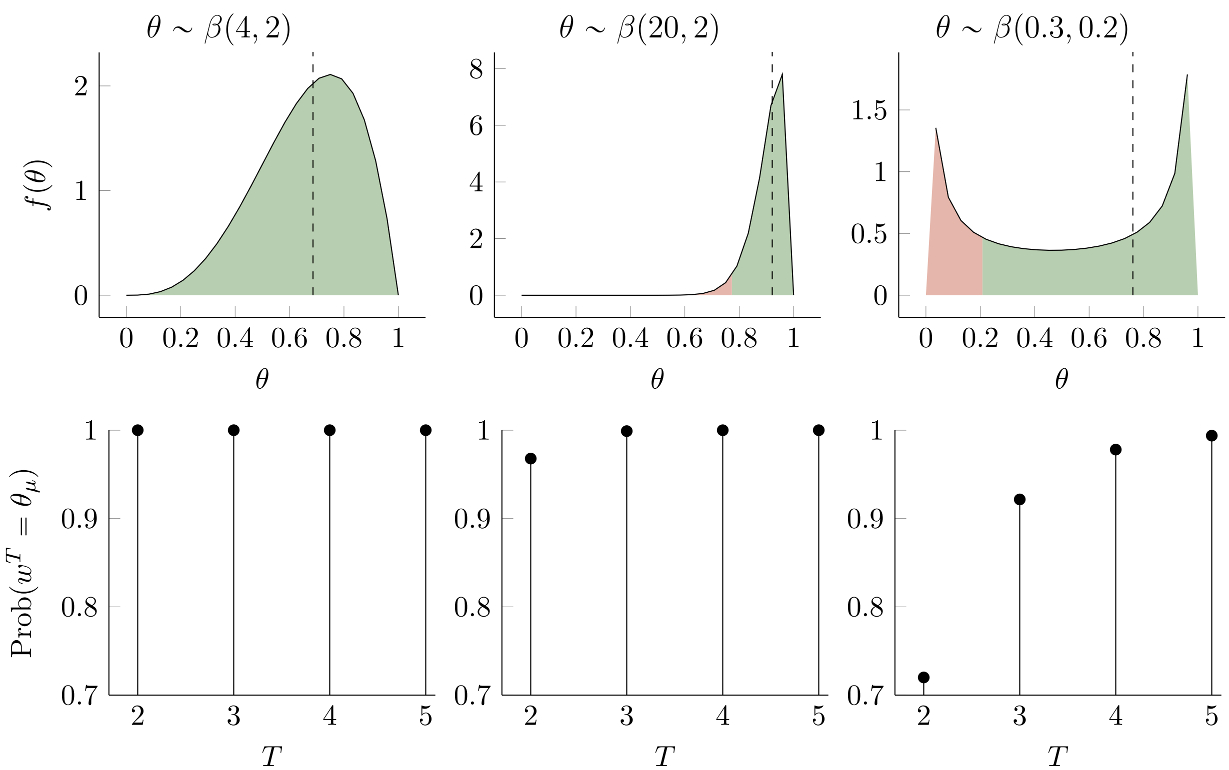

Figure 1: The first row depicts the optimal proposals in rounds $t \in \{1, 2, \ldots, T-1\}$ for different Beta distributions of the types. All types positioned in the green regions of the distribution propose the Condorcet winner, and those in the red regions propose their own type. The dashed line marks the position of the Condorcet winner. The second row shows the probability of the Condorcet winner being implemented for $T \in \{2, \ldots, 5\}$ for each of the three distributions of the types. Note that if the distribution is sufficiently symmetric, as in the left plot, all types propose the Condorcet winner.

It follows from the proposers' equilibrium strategy that for highly skewed distributions of the types and a status quo positioned at the tail of the distribution, the final winner might also be at the tail, though this scenario becomes less likely with more voting rounds. Thus, the probability of implementing the Condorcet winner rapidly approaches one within a few rounds, as illustrated in Figure 1. Specifically, when $\theta_\mu \geqslant 0.5$, we have $\text{Prob}(w^T = \theta_\mu) = 1 - F(\underline{\theta})^{T-1} \geqslant 1 - F(\theta_\mu)^{T-1}$. Using $1 - F(\theta_\mu)^{T-1}$ as a lower bound, it follows that the probability of implementing the Condorcet winner within six voting rounds exceeds 96.8%, regardless of the specific distribution of types.

This proposal strategy is supported by a truthful voting behavior of sophisticated voters maximizing their continuation utility. However, as we show in Lemma 3 in the Appendix, it is also sustained if voters are truthful and myopic, i.e. vote for the more preferred alternative in the current round. Intuitively, this is the case because there is always a majority of voters for which the two strategies coincide, given the optimal strategy of the proposers.

As we demonstrate in the following proposition, if the distribution is sufficiently symmetric or the initial status quo is sufficiently balanced, the VRP procedure effectively implements the socially optimal alternative in only two rounds of voting.

Proposition 1. *Voting with Random Proposers implements the Condorcet winner in $T = 2$ rounds for any status quo $q^1 \in [\underline{\theta}, \bar{\theta}]$ and any proposer's type $\theta_{s^1} \in [\underline{\theta}, \bar{\theta}]$, presupposed that the distribution of the voters' types F is such that*

$$
\begin{aligned}
&Var(\theta) \geqslant \theta_\mu^2 - \mathbb{E}(\theta)^2 && \text{if } \theta_\mu \geqslant 0.5, \\
&Var(\theta) \geqslant (1-\theta_\mu)^2 - (1-\mathbb{E}(\theta))^2 && \text{if } \theta_\mu < 0.5.
\end{aligned}
$$

Moreover, the voting procedure is strategy-proof.

Proof. See Appendix B. $\square$

Proposition 1 establishes the conditions under which the Condorcet winner can be implemented in two rounds. In particular, implementation is guaranteed when the distribution is not highly asymmetric, a condition that holds when the median and the mean are sufficiently close, regardless of the status quo and the proposer's type. Intuitively, in such cases, all types prefer to propose the Condorcet winner instead of their own peak since otherwise a next proposer may be able to implement a policy on the other side of the distribution of peaks. The threat of a polarized outcome tomorrow induces moderation in today's policies. Note that proposing the Condorcet winner is a weakly dominant strategy even in a polarized society, as long as the distribution is sufficiently symmetric. Additionally, we show that the Condorcet winner can be implemented in two rounds for arbitrary distributions of voter preferences, provided that the status quo or the proposer's type in the first round is not extreme.

5 Discussion and Outlook

We demonstrate that incorporating random proposers into an iterative majority voting process efficiently implements the socially optimal alternative within a few rounds. Furthermore, if individual utility-maximizing alternatives are sufficiently symmetrically distributed or the status quo is balanced, the VRP procedure selects the Condorcet winner in only two rounds.

We use a simple framework in which the distribution of peaks of the individual preferences is public knowledge. Even if the distribution is unknown, our results would imply that each proposer would like to guess the position of the Condorcet winner and propose it, unless the proposer believes to have a peak in the tail of the distribution. Therefore, the intermediate winners would also converge to the Condorcet winner, but possibly in more rounds.

It is well known that a Condorcet winner might not exist when preferences violate single-peakedness and pairwise voting procedures might exhibit cycles. Voting with Random Proposers offers a probabilistic solution to resolving such cycles. Future research should build on the work of Airiau and Endriss (2009), who show that the proposal dynamic resembles a Markov chain, and explore the convergence and strategic incentives of the players.

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Notation

Variable	Explanation
$[0, 1]$	Policy space
θ	Type of an agent in $[0, 1]$, representing his/her peak of preferences
F	Cumulative distribution function of types
f	Density function of types
θ_μ	Median of the type distribution
$\mathbb{E}(\theta)$	Mean of the type distribution
$\mathrm{Var}(\theta)$	Variance of the type distribution
$\beta(\alpha, \beta)$	Beta distribution with parameters α and β
$u_\theta(x)$	Utility of type θ for policy x
$c(x)$	Reflection of x around θ_μ, truncated to $[0, 1]$
$\underline{\theta}$	Lower threshold type
$\bar{\theta}$	Upper threshold type
T	Fixed number of voting rounds
q_t	Status quo in round t; $q_{t+1} = w_t$
p_t	Proposal made in round t by the randomly drawn proposer
w_t	Winner of the pairwise majority vote in round t
θ_{s^t}	Type of the proposer drawn in round t

A Appendix: Proof of Theorem 1

Recall that $c(x)$ denotes the farthest positioned alternative in $[0, 1]$ that is at least as preferred as x. It is given by the symmetric point of x with respect to the Condorcet winner, θ_μ, i.e., $c(x) := \min\{\max\{2\theta_\mu - x, 0\}, 1\}$. We solve the sequential game through backward induction starting at round T.

Optimal proposal in round T

Note that in the last round of voting T, the voters vote truthfully independent of the proposed alternatives. This is due to the fact that the procedure is a simple majority voting between two alternatives and the voters have single-peaked preferences. Thus, the winning alternative from the last pairwise vote $w(p^T, q^T)$ for $q^T \leqslant \theta_\mu$ is given by

$$(1) \qquad w(p^T, q^T) = \begin{cases} p^T & \text{if } p^T \in [q^T, c(q^T)], \\ q^T & \text{if } p^T \in [0, q^T) \cup (c(q^T), 1]. \end{cases}$$

The peak of the preference of the agenda-setter at round T is given by θ_{s^T}. Consider the optimization problem of the agenda-setter in round T:

$$\underset{p^T \in [q^T, c(q^T)]}{\arg\max} \; -(p^T - \theta_{s^T})^2.$$

Note that the proposer is not constrained about the position of the proposal in the interval $[0, 1]$ by the voting procedure, but he/she would anticipate that proposing an alternative in $[0, q^T)$ or $(c(q^T), 1]$ would lose the pairwise vote and cannot be utility-improving. Thus, the optimal proposal and final winner are given by

$$p^T \in \begin{cases} [0, q^T] & \text{if } \theta_{s^T} \in [0, q^T], \\ \{\theta_{s^T}\} & \text{if } \theta_{s^T} \in [q^T, c(q^T)], \\ \{c(q^T)\} & \text{else.} \end{cases} \qquad w(p^T, q^T) = \begin{cases} q^T & \text{if } \theta_{s^T} \in [0, q^T], \\ \theta_{s^T} & \text{if } \theta_{s^T} \in [q^T, c(q^T)], \\ c(q^T) & \text{else.} \end{cases}$$

Note that whenever $\theta_{s^T} \in [0, q^T)$, the winner is q^T, hence proposing θ_{s^T} is a weakly dominant strategy for $\theta_{s^T} \in [0, c(q^T)]$. We obtain an analogous result for the case $q^T > \theta_\mu$ and provide the final solution below:

$$p^T = \begin{cases} \min\{\theta_{s^T}, c(q^T)\} & \text{if } q^T \leqslant \theta_\mu, \\ \max\{\theta_{s^T}, c(q^T)\} & \text{if } q^T > \theta_\mu. \end{cases} \quad w(p^T, q^T) = \begin{cases} \min\{q^T, c(q^T)\} & \text{if } \theta_{s^T} < \min\{q^T, c(q^T)\}, \\ \max\{q^T, c(q^T)\} & \text{if } \theta_{s^T} > \max\{q^T, c(q^T)\}, \\ \theta_{s^T} & \text{else.} \end{cases}$$

Optimal proposal in round $T - 1$

We now consider the optimal proposal and voting at round $T - 1$. We organise the proof in the following way. First, we derive the winner of round $T - 1$ that would maximize the expected utility of the proposer, which, as we show, is either the proposer's own peak or the Condorcet winner. Then, we determine the optimal proposals in round $T - 1$. Finally, we show that the majority of the strategic voters would vote in favour of the proposal in Lemma 1.

We now state the expected utility of the proposer $\theta_{s^{T-1}}$ when proposing w in round $T-1$. To ease the notation we denote the proposers type $\theta_{s^{T-1}} = \theta_s$ in this subsection:

$$\mathrm{EU}_s(w) = \begin{cases} \int_0^w u_{\theta_s}(w) f(v) dv + \int_w^{c(w)} u_{\theta_s}(v) f(v) dv + \int_{c(w)}^1 u_{\theta_s}(c(w)) f(v) dv & \text{if } w \leqslant \theta_\mu, \\ \int_0^{c(w)} u_{\theta_s}(c(w)) f(v) dv + \int_{c(w)}^w u_{\theta_s}(v) f(v) dv + \int_w^1 u_{\theta_s}(w) f(v) dv & \text{if } w > \theta_\mu, \end{cases}$$

which can be simplified as follows:

$$\mathrm{EU}_s(w) = \begin{cases} u_{\theta_s}(w) F(w) + \int_w^{c(w)} u_{\theta_s}(v) f(v) dv + u_{\theta_s}(c(w))(1 - F(c(w))) & \text{if } w \leqslant \theta_\mu, \\ u_{\theta_s}(c(w)) F(c(w)) + \int_{c(w)}^w u_{\theta_s}(v) f(v) dv + u_{\theta_s}(w)(1 - F(w)) & \text{if } w > \theta_\mu. \end{cases}$$

Each of the three terms in the expected utility function corresponds to the possible winners in round T, depending on the proposer's type in round T. In the case when $w \leqslant \theta_\mu$ and for

$\theta_{s^T} \in [0, w^{T-1})$, the final winner is $w^T = q^T = w^{T-1}$. For $\theta_{s^T} \in [w^{T-1}, c(w^{T-1})]$, the final winner will be θ_{s^T}. Finally, for $\theta_{s^T} \in (c(w^{T-1}), 1]$, the final winner is $c(w^{T-1})$. The analogous argument holds for the case when $w \geqslant \theta_\mu$.

We proceed by finding the utility-maximizing winner considering the following cases.

Case 1: $w^{T-1} \in [0, 2\theta_\mu - 1]$. Thus, $c(w^{T-1}) = 1$. The first order condition is given by

$$u_{\theta_s}(w)'F(w) + u_{\theta_s}(w)f(w) - u_{\theta_s}(w)f(w) = u_{\theta_s}(w)'F(w) = 0.$$

Observe that $\mathrm{EU}_s(w)$ is monotonically increasing for $\theta_s \geqslant \max\{2\theta_\mu - 1, 0\}$. If $\theta_s < \max\{2\theta_\mu - 1, 0\}$, we have $u_{\theta_s}(w)' = 0$ when $w^{T-1} = \theta_s$. Thus, $\mathrm{EU}_s(w)$ has a maximum at $w^{T-1} = \theta_s$ as $u_{\theta_s}(w)' = 2(\theta_s - w)$ and

$$w^{T-1} = \min\{2\theta_\mu - 1, \theta_s\}.$$

Case 2: $w^{T-1} \in (2\theta_\mu - 1, \theta_\mu]$. By definition, $c(w^{T-1}) = 2\theta_\mu - w^{T-1}$. The FOC is given by[8]

$$\begin{aligned} u_{\theta_s}(w)'F(w) + & u_{\theta_s}(w)f(w) - u_{\theta_s}(c(w))f(c(w)) - u_{\theta_s}(w)f(w) - \\ & - u_{\theta_s}(c(w))'(1 - F(c(w))) + u_{\theta_s}(c(w))f(c(w)) = 0, \\ u_{\theta_s}(w)'F(w) - & u_{\theta_s}(c(w))'(1 - F(c(w))) = 0. \end{aligned}$$

Observe that if $w = \theta_\mu$, then $c(w) = \theta_\mu$ and the equation is satisfied. Hence, $w^{T-1} = \theta_\mu$.

Case 3: $w^{T-1} \in [\theta_\mu, 2\theta_\mu)$. We have $c(w^{T-1}) = 2\theta_\mu - w^{T-1}$. The FOC is given by

$$\begin{aligned} -u_{\theta_s}(c(w))'F(c(w)) - & u_{\theta_s}(c(w))f(c(w)) + u_{\theta_s}(w)f(w) + u_{\theta_s}(c(w))f(c(w)) + \\ & + u_{\theta_s}(w)'(1 - F(w)) - u_{\theta_s}(w)f(w) = 0, \\ -u_{\theta_s}(c(w))'F(c(w)) + & u_{\theta_s}(w)'(1 - F(w)) = 0. \end{aligned}$$

Thus, the utility maximizing w is equal to the Condorcet winner.

[8] One can also verify the second-order conditions.

Case 4: $w^{T-1} \in [2\theta_\mu, 1]$. Thus, by definition $c(w^{T-1}) = 0$. The FOC is given by

$$u_{\theta_s}(w)'(1 - F(w)) + u_{\theta_s}(w)f(w) - u_{\theta_s}(w)f(w) = u_{\theta_s}(w)'(1 - F(w)) = 0.$$

Observe that $\mathrm{EU}_s(w)$ is monotonically decreasing for $\theta_s \leqslant \min\{2\theta_\mu, 1\}$. If $\theta_s > \min\{2\theta_\mu, 1\}$, the expected utility is maximized at $w^{T-1} = \theta_s$. Thus,

$$w^{T-1} = \max\{2\theta_\mu, \theta_s\}.$$

Thus, the utility-maximizing winner of round $T - 1$ is

$$w^{T-1} \in \{\min\{2\theta_\mu - 1, \theta_{s^{T-1}}\}, \theta_\mu, \max\{2\theta_\mu, \theta_{s^{T-1}}\}\}.$$

We organise the rest of the proof in a series of lemmas:

1. we identify the interval of types of proposers who have a higher expected utility from θ_s winning round $T - 1$ instead of θ_μ when $\theta_s < 2\theta_\mu - 1$ or $\theta_s > 2\theta_\mu$,
2. we show that all such types of proposers prefer a winner within the interval over θ_μ,
3. truthful voting behavior is the optimal strategy.

In Lemma 3, we show that these voting strategies are equivalent in terms of outcomes to the voting behavior of myopic voters, who simply vote for the preferred alternative in a given round.

Lemma 1. *If $\theta_s \in [0, \underline{\theta}] \cup [\bar{\theta}, 1]$, then $EU_s(\theta_s) \geqslant u_{\theta_s}(\theta_\mu)$, and if $\theta_s \in [\underline{\theta}, \bar{\theta}]$, then $EU_s(\theta_s) \leqslant u_{\theta_s}(\theta_\mu)$.*

Proof. Assume that the distribution F is such that $\theta_\mu > 0.5$. We begin the proof by comparing the expected utility for type θ_s when the winner of round $T - 1$ is θ_s vs. the median peak for all types $\theta_s < 2\theta_\mu - 1$.

$$\begin{aligned}\mathrm{EU}_s(\theta_s) - u_{\theta_s}(\theta_\mu) &= u_{\theta_s}(\theta_s)F(\theta_s) + \int_{\theta_s}^{c(\theta_s)} u_{\theta_s}(v)f(v)dv + u_{\theta_s}(c(\theta_s))(1 - F(c(\theta_s))) - u_{\theta_s}(\theta_\mu) \\ &= \int_{\theta_s}^{1} u_{\theta_s}(v)f(v)dv - u_{\theta_s}(\theta_\mu),\end{aligned}$$

since $u_{\theta_s}(\theta_s) = 0$ and $c(\theta_s) = 1$. In order to find the type for which the above expression is positive, we will first show that the function $\mathrm{EU}_s(\theta_s) - u_{\theta_s}(\theta_\mu)$ is monotonically decreasing in θ_s. Differentiating the right-hand side of the above equality w.r.t. θ_s gives

$$2\left(\theta_s F(\theta_s) + \int_{\theta_s}^{1} v f(v)dv - \theta_\mu\right). \tag{2}$$

For $\theta_s = 0$, the above expression (2) is negative since the expected value is smaller than the median whenever $\theta_\mu > 0.5$ by definition of F. It is also monotonically increasing in θ_s, which is bounded from above by $2\theta_\mu - 1$. Thus, we need to show that

$$(2\theta_\mu - 1)F(2\theta_\mu - 1) + \int_{2\theta_\mu - 1}^{1} v f(v)dv - \theta_\mu \leqslant 0.$$

Doing integration by parts on the left-hand side of the above inequality and then rewriting the obtained term yields:

$$\begin{aligned}&(2\theta_\mu - 1)\,F(2\theta_\mu - 1) + \int_{2\theta_\mu - 1}^{1} v\,f(v)\,dv - \theta_\mu = 1 - \theta_\mu - \int_{2\theta_\mu - 1}^{1} F(v)\,dv \\ &= 1 - \theta_\mu - \int_0^{1-\theta_\mu} \big(F(\theta_\mu + t) + F(\theta_\mu - t)\big)\,dt = \int_0^{1-\theta_\mu} \Big(1 - F(\theta_\mu + t) - F(\theta_\mu - t)\Big)\,dt \leqslant 0,\end{aligned}$$

where the non-positivity of the last term follows from the skewness condition on the distribution F and the assumption that $\theta_\mu > 0.5$ (which forces the sign in the condition to be negative). Therefore, we have shown that $\mathrm{EU}_s(\theta_s) - u_{\theta_s}(\theta_\mu)$ is monotonically decreasing in θ_s. Recall the definition of the lower threshold type $\underline{\theta} = \max\{0, \{\theta < 2\theta_\mu - 1 : \int_\theta^1 u_\theta(v)f(v)dv = u_\theta(\theta_\mu)\}\}$. Thus, $\mathrm{EU}_{\underline{\theta}}(\underline{\theta}) - u_{\underline{\theta}}(\theta_\mu) = 0$ and $\forall \theta_s \in [0, \underline{\theta}]$, proposing the own type is associated with higher expected utility than proposing θ_μ. Note that for $\theta_s \in [2\theta_\mu - 1, 2\theta_\mu]$, our previous analysis shows that the expected utility is maximized at $p^{T-1} = \theta_\mu$.

We redo the analysis for distributions with $\theta_\mu \leqslant 0.5$ to obtain the final result; consider a distribution F with $\theta_\mu \leqslant 0.5$. Then we are interested in right-extreme types with $\theta_s > 2\theta_\mu$, where $c(\theta_s) = 0$. Using the previously obtained form of EU for $w = \theta_s > \theta_\mu$ we obtain $(u_{\theta_s}(\theta_s) = 0)$

$$EU_{\theta_s}(\theta_s) - u_{\theta_s}(\theta_\mu) = \int_0^{\theta_s} u_{\theta_s}(v) f(v) dv - u_{\theta_s}(\theta_\mu).$$

Analogous to the $\theta_\mu > 0.5$ case, we differentiate that term w.r.t. θ_s:

$$\frac{d}{d\theta_s}\Big(EU_{\theta_s}(\theta_s) - u_{\theta_s}(\theta_\mu)\Big) = 2\bigg(\int_0^{\theta_s} v f(v)\, dv + \theta_s(1 - F(\theta_s)) - \theta_\mu\bigg).$$

Notice that the term in the bracket is nondecreasing in θ_s, in particular on our interval of consideration $[2\theta_\mu, 1]$. Hence considering the term evaluated in $2\theta_\mu$ is sufficient.

$$\begin{aligned}\Big(EU_{\theta_s} - u_{\theta_s}(\theta_\mu)\Big)'_{\theta_s = 2\theta_\mu} &= \int_0^{2\theta_\mu} v f(v)\, dv + 2\theta_\mu(1 - F(2\theta_\mu)) - \theta_\mu \\ &= \int_0^{\theta_\mu} \Big(1 - F(\theta_\mu + t) - F(\theta_\mu - t)\Big)\, dt \;\geqslant\; 0,\end{aligned}$$

using the skewness condition. Hence we conclude that $EU_{\theta_s}(\theta_s) - u_{\theta_s}(\theta_\mu)$ is increasing in θ_s on $[2\theta_\mu, 1]$. Taking a second look at the definition

$$\overline{\theta} := \min\Big\{1,\ \{\theta > 2\theta_\mu :\ \int_0^{\theta} u_\theta(v) f(v)\, dv = u_\theta(\theta_\mu)\}\Big\},$$

we have equality at $\theta_s = \overline{\theta}$ and therefore

$$EU_{\theta_s}(\theta_s) - u_{\theta_s}(\theta_\mu) \geqslant 0 \qquad \text{for all } \theta_s \in [\overline{\theta}, 1].$$

The interior part argument ($\theta_s \in [\underline{\theta}, \overline{\theta}]$, with $\underline{\theta} = 0$ here) is identical to the > 0.5 case:

using the interior derivative

$$EU'_{\theta_s}(w) = \begin{cases} 2\Big[F(w)\,(\theta_s - w) + \big(1 - F(c(w))\big)\,\big(c(w) - \theta_s\big)\Big] & \text{if } w < \theta_\mu, \\ 2\Big[F\big(c(w)\big)\,(\theta_s - c(w)) + \big(1 - F(w)\big)\,(w - \theta_s)\Big] & \text{if } w > \theta_\mu, \end{cases}$$

one checks $EU'_{\theta_s}(w) \geqslant 0$ for $w < \theta_\mu$, $EU'_{\theta_s}(w) \leqslant 0$ for $w > \theta_\mu$, hence $EU_{\theta_s}(w)$ is maximized at $w = \theta_\mu$ and $EU_{\theta_s}(\theta_s) \leqslant u_{\theta_s}(\theta_\mu)$.

□

Next, we show that for sufficiently extreme types in $[0, \underline{\theta}] \cup [\bar{\theta}, 1]$, a winner that is within the same bound is more preferred than θ_μ.

Lemma 2. *For all $\theta_s \in [0, \underline{\theta}] \cup [\bar{\theta}, 1]$, $EU_s(w) \geqslant u_{\theta_s}(\theta_\mu)$ for all $w \in [\theta_s, \underline{\theta}] \cup [\bar{\theta}, \theta_s]$.*

Proof. Assume that the distribution F is such that $\theta_\mu > 0.5$ and $\underline{\theta} > 0$. First note that $\mathrm{EU}_{\theta_s}(w) - u_{\theta_s}(\theta_\mu)$ is monotone decreasing for all $w \in [\theta_s, \underline{\theta}]$. Thus, it suffices to show:

$$EU_{\theta_s}(\underline{\theta}) - u_{\theta_s}(\theta_\mu) = u_{\theta_s}(\underline{\theta})F(\underline{\theta}) + \int_{\underline{\theta}}^{1} u_{\theta_s}(v)f(v)dv - u_{\theta_s}(\theta_\mu) \geqslant 0 \tag{3}$$

for all $\theta_s \in [0, \underline{\theta}]$. First, observe that the type $\theta_s = \underline{\theta}$ is indifferent between proposing $\underline{\theta}$ and θ_μ (following from the definition of $\underline{\theta}$). Thus, if we can show that the function is monotonically decreasing w.r.t. θ_s, we would prove the claim. Differentiating the left-hand side w.r.t. θ_s gives

$$2\left(\underline{\theta}F(\underline{\theta}) + \int_{\underline{\theta}}^{1} vf(v)dv - \theta_\mu\right).$$

For $\underline{\theta} = 0$, the expression is negative since the expected value is smaller than the median whenever $\theta_\mu > 0.5$ by definition of F. It is also monotonically increasing in $\underline{\theta}$, which is bounded from above by $2\theta_\mu - 1$. Hence, it suffices to prove that $(2\theta_\mu - 1)\,F(2\theta_\mu - 1) + \int_{2\theta_\mu - 1}^{1} v\,f(v)\,dv - \theta_\mu \leqslant 0$. But we already showed this in the proof of Lemma 1, using integration by parts and the skewness condition on the distribution F.

Thus, we can conclude that $\underline{\theta}F(\underline{\theta}) + \int_{\underline{\theta}}^1 vf(v)dv - \theta_\mu \leqslant 0$ and therefore, $\forall \theta_s \in [0, \underline{\theta}]$, proposing θ_s is associated with higher expected utility than proposing θ_μ.

We shall repeat the analogous analysis for distributions with $\theta_\mu \leqslant 0.5$ to obtain the final result. Notice that for $\theta_\mu \leqslant 0.5$, $\underline{\theta} = 0$, so we only consider $\theta_s \in [\overline{\theta}, 1]$. Fix $\theta_s \geqslant \overline{\theta}$ and $w \in [\overline{\theta}, \theta_s]$. We split the argument into two claims to make it more legible.

First, we claim that $EU_{\theta_s}(w) \geqslant EU_{\theta_s}(\overline{\theta})$ for all $w \in [\overline{\theta}, \theta_s]$. Since $w \geqslant 2\theta_\mu$, $c(w) = 0$ and

$$EU_{\theta_s}(w) = \int_0^w u_{\theta_s}(v)f(v)\,dv + u_{\theta_s}(w)\big(1 - F(w)\big).$$

Taking the derivative w.r.t. w gives us

$$EU'_{\theta_s}(w) = -2(w - \theta_s)\,(1 - F(w)),$$

which is $\geqslant 0$ on $[\overline{\theta}, \theta_s]$. This tells us that $EU_{\theta_s}(w) \geqslant EU_{\theta_s}(\overline{\theta})$.

Second, we claim that for all $\theta_s \geqslant \overline{\theta}$, $EU_{\theta_s}(\overline{\theta}) \geqslant u_{\theta_s}(\theta_\mu)$.

Now, we consider $K(\theta_s) := EU_{\theta_s}(\overline{\theta}) - u_{\theta_s}(\theta_\mu)$ as a function of θ_s. First, notice that $K(\overline{\theta}) = 0$. Taking the derivative w.r.t. θ_s gives

$$K'(\theta_s) = 2\left(\int_0^{\overline{\theta}} vf(v)\,dv + \overline{\theta}(1 - F(\overline{\theta})) - \theta_\mu\right) = 2\left(\phi(\overline{\theta}) - \theta_\mu\right),$$

using a helper-function $\phi(x) := \int_0^x vf(v)\,dv + x(1 - F(x))$, which is nondecreasing since $\phi'(x) = 1 - F(x) \geqslant 0$. Hence, evaluating $2\left(\phi(2\theta_\mu) - \theta_\mu\right)$ gives a lower bound on the value of $K'(\theta_s)$:

$$2\left(\int_0^{2\theta_\mu} vf(v)\,dv + 2\theta_\mu(1 - F(2\theta_\mu)) - \theta_\mu\right) = 2\int_0^{\theta_\mu}\big(1 - F(\theta_\mu + t) - F(\theta_\mu - t)\big)\,dt \;\geqslant\; 0,$$

by the skewness condition, and thus $K'(\theta_s) \geqslant 0$. Using that $K(\overline{\theta}) = 0$, we get for all $\theta_s \geqslant \overline{\theta}$

that $K(\theta_s) \geqslant 0$, or equivalently $EU_{\theta_s}(\overline{\theta}) \geqslant u_{\theta_s}(\theta_\mu)$.

Combining the two claims now gives $EU_{\theta_s}(w) \geqslant EU_{\theta_s}(\overline{\theta}) \geqslant u_{\theta_s}(\theta_\mu)$ for all $\theta_s \in [\overline{\theta}, 1]$ and $w \in [\overline{\theta}, \theta_s]$. $\square$

The weakly dominant strategy for the agenda-setter in round $T-1$ is therefore:

$$(4) \qquad p^{T-1} = \begin{cases} \theta_{s^{T-1}} & \text{if } \max\{q^{T-1}, \theta_{s^{T-1}}\} < \underline{\theta} \text{ or } \min\{q^{T-1}, \theta_{s^{T-1}}\} > \bar{\theta}, \\ \theta_\mu & \text{else.} \end{cases}$$

Finally, we show that this optimal proposal can be sustained by truthful voting behavior. We also show that the outcome is equivalent to myopic voting, i.e. when voters simply vote for their preferred alternative in a given round.

Lemma 3. *Sophisticated and myopic voting are equilibrium strategies under the proposer's strategy shown in Equation (4).*

Proof. Suppose first that $p^{T-1} = \theta_\mu$. Let the distribution of the types is such that $\theta_\mu > 0.5$. We know from Lemma 1 that all types $\theta_s \notin [0, \underline{\theta}] \cup [\bar{\theta}, 1]$ would prefer θ_μ to be the winner of round $T-1$ than their own type. Thus, the majority of voters would vote for θ_μ in equilibrium. Since by definition $F(\theta_\mu) = 0.5$, the mass of voters who prefer θ_μ over q^{T-1} is sufficient to win the election even if voters are myopic. We can make the analogous argument if the distribution of the types is such that $\theta_\mu \leqslant 0.5$.

Next, consider the strategic voting behavior when $p^{T-1} = \theta_{s^{T-1}}$. Suppose that F is such that $\theta_\mu > 0.5$, hence $\max\{q^{T-1}, \theta_{s^{T-1}}\} < \underline{\theta}$. Note that $c(\max\{q^{T-1}, \theta_{s^{T-1}}\}) = 1$. In that case, all voters with a type in $[\max\{q^{T-1}, \theta_{s^{T-1}}\}, 1]$ strictly prefer $\max\{q^{T-1}, \theta_{s^{T-1}}\}$, since they anticipate that the final winner will be in the same interval as opposed to $[\min\{q^{T-1}, \theta_{s^{T-1}}\}, 1]$ if they support $\min\{q^{T-1}, \theta_{s^{T-1}}\}$. Therefore, $\max\{q^{T-1}, \theta_{s^{T-1}}\}$ would win the vote in round $T-1$. Recall that proposers of type $\theta_{s^{T-1}}$ have a higher expected utility from $\max\{q^{T-1}, \theta_{s^{T-1}}\}$ than θ_μ as shown in Lemma 2, so they would still propose their type even if it loses against the status quo.[9] We obtain the analogous result for distributions with $\theta_\mu \leqslant 0.5$. $\square$

Optimal proposal in rounds $t \in \{1, 2, \ldots, T-2\}$

Note that if the distribution of the types is such that $\underline{\theta} = 0$ and $\bar{\theta} = 1$, it is optimal for all types to propose the Condorcet winner, based on our previous analysis. Suppose now that the distribution is such that $\theta_\mu > 0.5$ and $\underline{\theta} > 0$. If $q^{T-2} > \underline{\theta}$, the majority of utility-maximizing (but also myopic) voters would not support a proposal $p^{T-2} < \underline{\theta}$, hence θ_μ will be proposed in the next round and will win the final round. Let $q^{T-2} \leqslant \underline{\theta}$ and assume that voters vote in a way that maximizes their continuation utility. Note that $c(q^{T-2}) = 1$ by definition. Therefore, the optimal proposal in round $T-1$ is as follows:

$$p^{T-1} = \begin{cases} \theta_{s^{T-1}} & \text{if } \max\{p^{T-2}, q^{T-2}, \theta_{s^{T-1}}\} < \underline{\theta}, \\ \theta_\mu & \text{else.} \end{cases}$$

Hence, in finding the optimal proposal in round $T-2$ we compare the expected utility of making a proposal below the threshold value with the utility from proposing the median type. This is the case because, if $p^{T-2} > \underline{\theta}$, then the expected utility of the agenda-setter is equal to the utility from the Condorcet winner, because it is going to be proposed in the next round and implemented in the final round. First note that for all $\theta_{s^{T-2}} \in [0, \underline{\theta}]$

$$\mathrm{EU}_{\theta_{s^{T-2}}}(\theta_{s^{T-2}}) - u_{\theta_{s^{T-2}}}(\theta_\mu) \geqslant \mathrm{EU}_{\theta_{s^{T-2}}}(\underline{\theta}) - u_{\theta_{s^{T-2}}}(\theta_\mu).$$

This is the case because, if all subsequent agenda-setters' types are smaller than $\underline{\theta}$, the final outcome is in the interval $[\max\{\theta_{s^{T-2}}, q^{T-2}\}, \underline{\theta}]$, and is thus more preferred than $\underline{\theta}$. If $\theta_{s^{T-1}} \leqslant \underline{\theta}$ and $\theta_{s^T} \geqslant \theta_{s^{T-1}}$, the final outcome will be in the interval $[\theta_{s^{T-1}}, 1]$, which will give the agenda-setter at $T-2$ a higher expected utility than by proposing $\underline{\theta}$, in which case the

[9] Note that this equilibrium strategy leads to the same outcome if voters follow a myopic truthful voting strategy.

final winner is in the interval $[\underline{\theta}, 1]$. Finally, if $\theta_{s^{T-1}} > \underline{\theta}$, the agenda-setter is indifferent between the two strategies, because θ_μ will be proposed and will win. Let us now derive the difference in expected utilities by proposing $\underline{\theta}$ vs. the Condorcet winner:

$$\begin{aligned}\mathrm{EU}_{\theta_{s^{T-2}}}(\underline{\theta}) - u_{\theta_{s^{T-2}}}(\theta_\mu) &= u_{\theta_{s^{T-2}}}(\underline{\theta})F(\underline{\theta})^2 + F(\underline{\theta})\int_{\underline{\theta}}^1 u_{\theta_{s^{T-2}}}(v)f(v)dv - u_{\theta_{s^{T-2}}}(\theta_\mu)F(\underline{\theta}) \\ &= F(\underline{\theta})(\mathrm{EU}_{\theta_{s^{T-1}}}(\underline{\theta}) - u_{\theta_{s^{T-1}}}(\theta_\mu)).\end{aligned}$$

The first term in the above expression refers to the case when all subsequent agenda-setters' types are in the interval $[0, \underline{\theta}]$ and hence, $\underline{\theta}$ is implemented. The second term refers to the case when $\theta_{s^{T-1}} \leqslant \underline{\theta}$ and $\theta_{s^T} \geqslant \underline{\theta}$ and the final winner is θ_{s^T}. Finally, if $\theta_{s^{T-1}} > \underline{\theta}$, the Condorcet winner is proposed in round $T-1$ and wins irrespective of the proposal in the last round. As we have shown in the proof of Lemma 2, $\mathrm{EU}_{\theta_{s^{T-1}}}(\underline{\theta}) - u_{\theta_{s^{T-1}}}(\theta_\mu) \geqslant 0$, and hence, the optimal $p^{T-2} = \theta_{s^{T-2}}$ whenever $\theta_{s^{T-2}} \leqslant \underline{\theta}$.

Next, suppose that $\theta_s \in [\underline{\theta}, \theta_\mu]$. We compare $\mathrm{EU}_{\theta_s}(\underline{\theta})$ against $u_{\theta_s}(\theta_\mu)$.

$$\mathrm{EU}_{\theta_s}(\underline{\theta}) - u_{\theta_s}(\theta_\mu) = u(\underline{\theta})F(\underline{\theta})^2 + F(\underline{\theta})\int_{\underline{\theta}}^1 u(v)f(v)dv - u(\theta_\mu)F(\underline{\theta}) \leqslant 0.$$

Note that $u(\underline{\theta}) < 0$ and $\int_{\underline{\theta}}^1 u(v)f(v)dv - u(\theta_\mu) < 0$ for $\theta_s \in (\underline{\theta}, \theta_\mu]$. Therefore, such agents are better off by proposing the Condorcet winner. Thus, the optimal proposal for all agenda-setters in round $T-2$ coincides with the one in round $T-1$. We can repeat the same argument for distributions with $\theta_\mu \leqslant 0.5$ and verify that the optimal proposal coincides with the one derived for round $T-1$.

Note that the argument generalizes to all rounds $t \in \{1, 2, \ldots, T-2\}$ since

$$\mathrm{EU}_{\theta_{s^t}}(\underline{\theta}) - u_{\theta_{s^t}}(\theta_\mu) = u_{\theta_{s^t}}(\underline{\theta})F(\underline{\theta})^{T-t} + F(\underline{\theta})^{T-t-1}\int_{\underline{\theta}}^1 u_{\theta_{s^t}}(v)f(v)dv - u_{\theta_{s^t}}(\theta_\mu)F(\underline{\theta})^{T-t-1}.$$

Finally, observe that Lemma 1 holds in that case as well, hence the voting procedure is strategy-proof.

In order to obtain the probability of implementing the Condorcet winner, observe that the only case in which it will not be proposed in the first $T-1$ rounds is when all types of proposers are in the interval $[0, \underline{\theta}]$ or $[\bar{\theta}, 1]$, depending on the skewness of the distribution. Moreover, no other alternative is proposed in the first $T-1$ rounds, which leads us to the final answer.

B Appendix: Proof of Proposition 1

Observe that if the distribution of preferences is such that $\underline{\theta} = 0$ when $\theta_\mu \geqslant 0.5$ and $\bar{\theta} = 1$ when $\theta_\mu < 0.5$, the optimal proposal at $T-1$ is always θ_μ. Therefore, we need that $\int_0^1 u_0(v)f(v)dv \leqslant u_0(\theta_\mu)$ when $\theta_\mu \geqslant 0.5$ and $\int_0^1 u_1(v)f(v)dv \leqslant u_1(\theta_\mu)$ when $\theta_\mu < 0.5$. Substituting the utility function, we obtain in the former case:

$$\int_0^1 v^2 f(v)dv \geqslant \theta_\mu^2,$$

which is equal to $\theta_\mu^2 - \mathbb{E}(\theta)^2 \leqslant \mathrm{Var}(\theta)$. Note that if the distribution is such that $\theta_\mu \geqslant 0.5$, this implies that $\theta_\mu > \mathbb{E}(\theta)$ and the inequality is not trivially satisfied. In the latter case, we simplify the inequality $\int_0^1 u_1(v)f(v)dv \leqslant u_1(\theta_\mu)$ to obtain:

$$\int_0^1 v^2 f(v)dv - \theta_\mu^2 \geqslant 2(\int_0^1 vf(v)dv - \theta_\mu)$$

$$\mathrm{Var}(\theta) + \mathbb{E}(\theta)^2 - \theta_\mu^2 \geqslant 2(\mathbb{E}(\theta) - \theta_\mu)$$

$$\mathrm{Var}(\theta) \geqslant (\mathbb{E}(\theta) - \theta_\mu)(2 - \mathbb{E}(\theta) - \theta_\mu)$$

If $\theta_\mu < 0.5$, then by definition of F, we have $\theta_\mu < \mathbb{E}(\theta)$. This implies that the above inequality is not trivially satisfied. Anticipating that θ_μ will be proposed at $T-1$ the latest, all previous agenda-setters cannot do better by making other proposals. Knowing that the Condorcet winner is going to be proposed at least at round $T-1$, voting truthfully is therefore a weakly dominant strategy for all voters.